EJERCICIOS DE TRIGONOMETRÍA

1.- Demuestra las siguientes igualdades:

a) $\dfrac{1+tg^2\alpha}{\cot g\alpha}=\dfrac{tg\alpha}{\cos^2\alpha}$

b) $tg\,2x=\dfrac{tgx\cdot(1+\cos 2x)}{\cos 2x}$

c) $\dfrac{tga+tgb}{\cot gb-\cot ga}=+tga\cdot tgb$

d) $tg\dfrac{a}{2}=\cos eca-\cot ga$

e) $tgx\cdot senx=\sec x-\cos x$

f) $tgA+tgB+tgC=tgA\cdot tgB\cdot tgC$
Si $A+B+C=180^{\circ}$

g) $\cos^4 x-sen^4x=2\cos^2 x-1$

h) $\dfrac{1}{\sec^2 x}=sen^2x\cdot\cos^2 x+\cos^4 x$

i) $\left(\cos ecx+\cot gx\right)\cdot(\cos ecx-\cot gx)=1$

j) $(\cos a+sena)^2=sen2a+1$

k) $tg(45^{\circ}+a)-tg(45^{\circ}-a)=2tg2a$

l) $sen^2\alpha-\cos^2\beta=sen^2\beta-\cos^2\alpha$

m) $\dfrac{sen(a+b)}{sen(a-b)}=\dfrac{tga\cdot\cot gb+1}{tga\cdot tgb-1}$

n) $\dfrac{tg\left(\dfrac{a+b}{2}\right)}{tg\left(\dfrac{a-b}{2}\right)}=\dfrac{sena+senb}{sena-senb}$

ñ) $\dfrac{tga+tgb}{\cot ga+\cot gb}=tga\cdot tgb$

o) $tgA\cdot tgB+tgB\cdot tgC+tgA\cdot tgC=1$
Si $A+B+C=90^{\circ}$

2.-Resuelve las siguientes ecuaciones trigonométricas:

$a)\cos^2 x-3sen^2x=0$

$Sol: x=\begin{cases}30^{\circ}+360^{\circ}k;150^{\circ}+360^{\circ}k\\210^{\circ}+360^{\circ}k;330^{\circ}+360^{\circ}k\end{cases};\forall k\in\mathbb{Z}$

$b)\cos 2x=1+4senx$

$Sol:180^{\circ}k;\forall k\in\mathbb{Z}$ (para k=cualquier número entero)

$c)tg^2x-3tgx+2=0$

$Sol: x=63,43^{\circ}+180^{\circ}k;\forall k\in\mathbb{Z}$

$d)sen5x+senx=0$

$Sol:\begin{cases}120^{\circ}k;60^{\circ}+120^{\circ}k;\\45^{\circ}+180^{\circ}k;135^{\circ}+180^{\circ}k\end{cases};\forall k\in\mathbb{Z}$

$e)tgx\cdot tg2x=1$

$Sol: x=\begin{cases}30^{\circ}+360^{\circ}k;150^{\circ}+360^{\circ}k\\210^{\circ}+360^{\circ}k;330^{\circ}+360^{\circ}k\end{cases};\forall k\in\mathbb{Z}$

$f)2senx=tgx$

$Sol: x=\begin{cases}0^{\circ}+360^{\circ}k;60^{\circ}+360^{\circ}k\\180^{\circ}+360^{\circ}k;300^{\circ}+360^{\circ}k\end{cases};\forall k\in\mathbb{Z}$

$g)2\cos^2 x+senx=1$

$Sol: x=\begin{cases}90^{\circ}+360^{\circ}k\\210^{\circ}+360^{\circ}k;\forall k\in\mathbb{Z}\\330^{\circ}+360^{\circ}k\end{cases}$

$h)3\cos x=2\sec x-5$

$Sol: x\approx\begin{cases}70,5^{\circ}+360^{\circ}k\\289,5^{\circ}+360^{\circ}k\end{cases};\forall k\in\mathbb{Z}$

$i) sen 2x \cdot \cos x = 6 sen^3 x$

$$Sol: x \approx \begin{cases} 0^o + 360^o k; 180^o + 360^o k \\ 30^o + 360^o k; 150^o + 360^o k \quad ; \forall k \in \mathbb{Z} \\ 210^o + 360^o k; 330^o + 360^o k \end{cases}$$

$j) \cos 8x + \cos 6x = 2 \cos 210^o \cos x$

$$Sol: x = \begin{cases} \frac{150^o + 360^o k}{7}; \frac{210^o + 360^o k}{7} \quad ; \forall k \in \mathbb{Z} \\ 90^o + 360^o k; 270^o + 360^o k \end{cases}$$

3.-Expresa $sen 3\alpha$ en función de $sen\alpha$

$Sol: sen 3\alpha = sen\alpha \cdot (3 - 4 sen^2 \alpha)$

4.-Halla $sen\left(\frac{\alpha}{2}\right), \cos\left(\frac{\alpha}{2}\right), tg\left(\frac{\alpha}{2}\right)$ sabiendo que $\cos\alpha = -\frac{1}{3}$ y que $\alpha \in \left[\pi, \frac{3\pi}{2}\right]$.

$Sol: \frac{\sqrt{6}}{3}, -\frac{\sqrt{3}}{3}, -\sqrt{2}$.

5.-A partir de las razones trigonométricas del ángulo de 45º, deduce el valor de $sen(292,5^o)$.

$Sol: -\frac{\sqrt{2+\sqrt{2}}}{2}$.

6.-Sabiendo que α y β pertenecen a dos cuadrantes no consecutivos y que $\cos\alpha = -\frac{1}{5}$ y $sen\beta = \frac{2}{3}$, halla $sen(\alpha + \beta)$, $\cos(\alpha - \beta)$, $sen 2\beta$, $\cos\left(\frac{\alpha}{2}\right)$.